2018 Fall EECS205003 Linear Algebra - Midterm 1 sol. Name: ID:

1. (a) *kuk* =*√*2, *kvk* =*√*2 and *θ* = 60*◦*

(b)

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1

20 1

2

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*−*1

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(c) *w* =

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*−*1 1

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(d) Let *q*(*x*) = *a*0 + *a*1*x* + *a*2*x*2 *∈ P*2

*< p, q >*= 2*a*0 + 3*a*1 + 3*a*2 = 0, *a*0 = *−*1, *a*1 = 0 and *a*2 = 4

No, *< p, q >6*= 0 so that *p*(*x*) isn’t orthogonal to *q*(*x*).

2. (a) A 2 *− D* plane, or linear combination of the columns of *A*

(b) All b *∈* C(*A*)*.*

(c) The first column or the second column. (Or the third column if the condition is written clearly.)

3. (a) Yes

(b) No, cause it doesn’t include 0

(c) Yes

(d) No, cause 0 is not included

(e) Yes

(f) Yes

4. (a) Step1: Subtract three times the first row of A from the second row, and subtract the first row from the third.

Step 2: Subtract twice the second row from the third.

A =

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1 2 1 3 7 6 1 4 8

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 *→*

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1 2 1 0 1 3 0 2 7

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 *→*

 

1 2 1 0 1 3 0 0 1

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= U

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

∴ L =

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1 0 0 3 1 0 1 2 1

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We solve Ax =

58 *−*12

 in two steps. First we solve Ly=b which in the case is 1

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1 0 0 3 1 0 1 2 1

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*y*1 *y*2 *y*3

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 =

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5

8

*−*12

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and we get y1 = 5; 15 + *y*2 = 8 so y2 = *−*7; and 5 *−* 14 + *y*3 = *−*3, so y3 = *−*3 Finally we solve Ux=y which in the case is

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1 2 1 0 1 3 0 0 1

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*x*1 *x*2 *x*3



=

 

5

*−*7 *−*3

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Here we have x3 = *−*3;*x*2 *−* 9 = *−*7, so x2 = 2; *x*1 + 4 *−* 3 = 5, so x1 = 4

(b) Here we have to reduce [L I] to [I *L−*1] by row operations,

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1 0 0 1 0 0 

3 1 0 0 1 0 1 2 1 0 0 1

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 *→*



1 0 0 1 0 0 

0 1 0 *−*3 1 0 0 2 1 *−*1 0 1



 *→*

1 0 0 1 0 0 

0 1 0 *−*3 1 0 0 0 1 5 *−*2 1



So *L−*1 = 

 

1 0 0 *−*3 1 0 5 *−*2 1

 

∴ *U−*1=



1 *−*2 5 0 1 *−*3 0 0 1



 by row operations [U I] to [I *U−*1].

Since A=LU and L and U are both invertible, we have *A−*1 = *U−*1*L−*1.

*A−*1 =

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1 *−*2 5 0 1 *−*3 0 0 1

 

 

1 0 0 *−*3 1 0 5 *−*2 1



 =



32 *−*12 5 

*−*18 7 *−*3 5 *−*2 1

 

(c) To convert to a matrix form, use the general format Ax= b:

2 3 6 15

*x y*

=

5 12

Subract three times the first row from the second row to get:

A=

2 3 6 15

*→ U* =

2 3 0 6

.

Doing the same to the right side b =(5,12) gives a new equation of the form Ux=c:

To solve our new equation,

2 3 0 6

*x y*

=

5 *−*3

6y = -3 *→* y = *−*12

2x+3y=5 *→* 2x+3(- 12)=5 *→* x =134

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(d) Let permutation matrix P =

0 1 0 1 0 0 0 0 1

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For any permutation matrix *P P T*, *Pij* =

(

1*,* if *i* = *j.*

0*,* if *i 6*= *j.*(1)

Only in the diagonals the above condition is satisfied , so this proves that P*PT* =I. 2

P*PT* =

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0 1 0 1 0 0 0 0 1

 

 

0 1 0 1 0 0 0 0 1



 =

 

1 0 0 0 1 0 0 0 1



 = I

(e) There are two 3\*3 permutation matrices,

i)

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0 0 1

1 0 0 0 1 0



 ii)

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0 1 0

0 0 1 1 0 0

 

5. (a) *l*21 = 1 *l*31 = *−*2 *l*32 = 1

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



*E*21 =

 

1 0 0 *−*1 1 0 0 0 1



 *E*31 =



1 0 0 0 1 0 2 0 1

 *E*32 =



1 0 0 0 1 0 0 *−*1 1



(b) *U* =

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1 *−*1 4

0 *−*1 *−*1 0 0 6



 x =



14 *−*2

 

(c) *L* = 

 

1 0 0 1 1 0 *−*2 1 1 



 c = 

 

*−*11 *−*13 8





 c = 

 

*−*11 *−*2

*−*12 

 

1 *−*1 4

0 *−*1 *−*1 0 0 6

 x =

*−*11

*−*2

*−*12

 x =

14 *−*2



6. (a) *M*2 =

*A*2 + *UV AU* + *UD*

*V A* + *DV V U* + *D*2

(b) *W A* = *Im* , *XA* + *InV* = 0 , *W U* = *Y* , *XU* + *InD* = *Z ··· A* is invertible

*··· W* = *A−*1, *X* = *−V A−*1, *Y* = *A−*1*U* , *Z* = *−V A−*1*U* + *D*

(c)

*Im Y* 0 *Z*

*M P N Q*

=

*Im* 0 0 *In*

*M* = *Im* and *N* = 0 (Can use simple matrix and Gauss-Jordan Elimination method to think)

*Im Y* 0 *Z*

*Im P* 0 *Q*

=

*Im* 0 0 *In*

so we can get *ImP* + *Y Q* = 0, *ZQ* = *In*

because the *Z* is invertible, *Q* = *Z−*1, *P* = *−Y Q* = *−Y Z−*1

*Im Y* 0 *Z*

*−*1

=

*Im −Y Z−*1 0 *Z−*1

use (b) can get :

*Im −A−*1*U*(*D − V A−*1*U*)*−*1

0 (*D − V A−*1*U*)*−*1

(d) use (b) can get :

*A−*1 0 *−V A−*1*In*

*A U V D*

=

*Im A−*1*U* 0 *D − V A−*1*U*

*A U V D*

use (c)

*−*1

=

*Im A−*1*U* 0 *D − V A−*1*U*

*−*1 *A−*1 0 *−V A−*1*In*

*A U V D*

*−*1

=

*Im −A−*1*U*(*D − V A−*1*U*)*−*1 0 (*D − V A−*1*U*)*−*1

*A−*1 0 *−V A−*1*In*

*A−*1 + *A−*1*U*(*D − V A−*1*U*)*−*1*V A−*1 *−A−*1*U*(*D − V A−*1*U*)*−*1

=

*−*(*D − V A−*1*U*)*−*1*V A−*1(*D − V A−*1*U*)*−*1 3

7. (a) To solve S, firstly we model S. The simplest one is to assume that *S* as a linear model, where *y* = *θaxa* + *θbxb* + *θcxc* with real-number parameters. Write down the equations for the three experiments in matrix form.

*θa* + 2*θb* + 3*θc* = 1

By samples, we can list up three equations

2*θa* + *θb* + 2*θc* = 1

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3*θa* + 2*θb* + *θc* = 1

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The matrix form is

1 2 3 2 1 2

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 

*θa θb*



 =

 

1 1

 

(b) By elimination

3 2 1

*θc*

1

 

1 2 3 2 1 2 3 2 1



row2 - 2 row1 *−−−−−−−−→*

 

1 2 3 0 *−*3 *−*4 3 2 1



row2 - 3 row1 *−−−−−−−−→*

 

1 2 3 0 *−*3 *−*4 0 *−*4 *−*8



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row3 - 4/3 row2 

*−−−−−−−−−−→*

1 2 3 0 *−*3 *−*4 0 0 *−*83

 

Then we have three elimination matrices

*E*21 =

 

1 0 0 *−*2 1 0 0 0 1



 *, E*31 =

 

1 0 0 0 1 0 *−*3 0 1



 *, E*32 =

 

1 0 0 0 1 0 0 *−*431



 *,*

Elimination matrix *E* is then



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*E* = *E*32*E*31*E*21 =

and decomposed matrix *L* and *U* are

1 0 0 *−*2 1 0 *−*13 *−*430

 

*L* =

 

1 0 0 2 1 0 3431



 *, U* =

 

1 2 3 0 *−*3 *−*4 0 0 *−*83

 

(c) Let *X* =

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1 2 3 2 1 2 3 2 1



, *θ* =

 

*θa θb θc*



 and y =

 

1 1 1

 





*EXθ* = *E*b *⇒ Uθ* =

1

*−*1 *−*23



 *⇒*

 

1 2 3 0 *−*3 *−*4 0 0 *−*83



 *θ ⇒*

*θc* = *−*38(*−−*23) = 14 *θb* =*−*1+4*θc*

*−*3 = 0

*θa* = 1 *−* 2*θb −* 3*θc* =14



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(d) Let matrix *X* =

*−*x*T*1 *− −*x*T*2 *− −*x*T*3 *−*

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

*xa*

 has rows representing sample input vector 

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*xb*

*xc*

If *X* has some rows that is linear dependent to others, but the corresponding outputs are not,

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then *S* has no solution. For example, change *X* to

2 4 6 1 2 3 3 2 1

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 but let *y* remain the same as

on the problem. Since some rows are linear dependent, this indicates after elimination # of pivots *<* 3. Then inverse of *X* does not exist and system *S* is not solvable. If *y* changes to 2*,* 1 and 3 in this case, *S* is still solvable but we cannot find the unique parameters.

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